CALCULATION OF THE UNCERTAINTY DUE TO SYSTEMATIC ERRORS OF TOTAL PRESSURE MEASUREMENT DURING FAST RESPONSE PROBES CALIBRATION

Milan Bulajić, Đorđe Čantrak, Dejan Ilić

Key words: Three component flow, pressure, multi-hole probe, FRAP, turbulence, measurement uncertainty.

ABSTRACT

Simultaneous measurement of 3D velocity and pressure fields, with high sampling rate, is a perpetual challenge. These complex flows exist in many technical systems, as well in nature. Turbulent swirling flow at the axial fan pressure side is one of the most demanding tasks. According to numerous authors, the occurrence of the turbulent swirling flow in ventilation systems is a source of energy loss, displacement of the designed fan duty point, and, as a result possible instability of the fan operation and reduction of the energy efficiency of the entire system.

Multi-hole probes are great tool for investigation of such complex flows. Recently, pressure transmitters have been integrated into the probe holder, so such probes have a fast response, i.e., a high sampling rate during measurements. The sampling rate depends on the geometry of the probe. The probe presented in this paper can sample several thousand samples per second, up to 10 kHz. This is of great importance, considering the existence of a very high level of turbulence in velocity and pressure.

The unsteadiness, non-homogeneity and three-component nature of such a velocity field, are requirements that lead to a probe geometry with a large number of holes with built-in high-frequency transmitters. This has led to the production of the FRAP (Fast Response Aerodynamic Probe), which has five holes and meets these criteria. Unlike standard probes (Pitot), FRAP probes need to be regularly calibrated.

The calibration process involves complex and time-consuming procedure of placing the probe in each of the working positions or angles of attack and collecting signals and values. The signals need to be processed, and subsequent calculations must be performed to determine the calibration coefficients. Additionally, uncertainties must be calculated, taking into account various factors that may affect the accuracy of the entire process.

INTRODUCTION

Testing the flow and velocity fields in the spaces before and after turbomachines presents a significant problem because turbomachines, with their rotating parts, cause large fluctuations in the flow field. This instability is caused by the interaction of the stator and rotor as well as secondary and vortex flows [1].
The methods of measuring these parameters and phenomena can be categorized into two groups: invasive and non-invasive (optical) methods. Optical methods include LDA (Laser Doppler Anemometry) and PIV (Particle Image Velocimetry). These methods allow for a detailed analysis of the velocity field, but they also have their drawbacks, as they require an optical opening at the measurement site, which can be complicated in some cases. Additionally, these methods cannot provide data on the pressure field.
Invasive methods, as the name suggests, require physically entering the flow section with some apparatus. This category includes classical probes, such as Pitot-Prandtl probes, as well as hot-wire anemometry (HWA) probes. Classical probes cannot provide enough data on the three-component characteristics of velocity and pressure, nor do they have sufficient sampling speed. HWA has different issues, as their construction is too fragile for operation in harsh working conditions, and calibration is required before each series of measurements [1].

These challenges have led to the development of multi-hole probes that have a high sampling speed to observe unsteady phenomena – FRAP (Fast Response Aerodynamic Probe). Although their development has been ongoing for decades, advancements were made possible by the significant development of semiconductor pressure sensors, and with the development of piezoresistive sensors, the frequency of data sampling has increased significantly.

The main advantage of FRAP over HWA probes is their wide sampling range, high signal-to-noise ratio, reliability, and robustness. On the other hand, their disadvantage is their bulkiness, which contributes to flow disturbance [2].

The probe discussed in this paper has the ability to sample up to 10 kHz, with a probe head diameter of 3 mm.

Figure 1 FRAP head close up [3]

The downside of reducing the size of the probe is that errors can occur during the manufacturing of the parts, manifesting as imperfections. For this reason, discrepancies arise between the results given by the model and those given by the fabricated part, precisely due to the mismatch between the fabricated probe geometry and the model [4].

Therefore, thorough calibration of each probe is necessary, and since such a probe additionally samples velocity and pressure values in all three dimensions, the device must be calibrated in all possible working positions. This is done by rotating the probe around two orthogonal axes. One is around the probe handle (α axis), and the other is in the right plane (β axis).

FRAP as its exit value gives out the value of voltage, therefore calculations have to be made to transform those values into desirable ones. Which are the three components of speed and the value of the static pressure inside of the stream. For each result, measurement uncertainty has to be taken into account. To do this, calculations have to be done taking into account each and every possible source of the forementioned uncertainty and adding them to get the total value of the uncertainty.

AERODYNAMIC CALIBRATION

For the calibration to take place, the probe has to be placed inside of a wind tunnel which has desirable traits, which are single dimensional speed characteristics and a low turbulence level (under 2%). This is achieved by fluid flow dampers which are placed inside sections in the wind tunnel. Such wind tunnels need to be checked and evaluated to be sure that they fulfill these requirements. During the calibration, additional measuring instruments are places next to the calibrating probe which check the parameters of the wind tunnel [4]. These parameters are later used for the calculations of the coefficients of the probe. A typical instrument used for this is a Pitot probe, which can measure the total pressure, and barometer is used to measure the static pressure, therefore the speed of the fluid can be calculated. Measurement standard for the calibration process is WMO-No. 8 [10], ISO 17713-1 [11] and ISO 3966 [12].

Each calibrating points starts by positioning the probe in a single position where the angle of attack is known and is a combination of two angles (α and β). The speed of the fluid which is impeding the probe is known and set beforehand. Values of voltage which are emitted by the probe's 5 sensors are collected and saved in the database for further calculations. The values of other measuring equipment are also saved and calculations can continue [3].

Besides angles α and β another angle combination is introduced which allows the probe to function in a wider range of angles of attack. These angles are φ and θ.

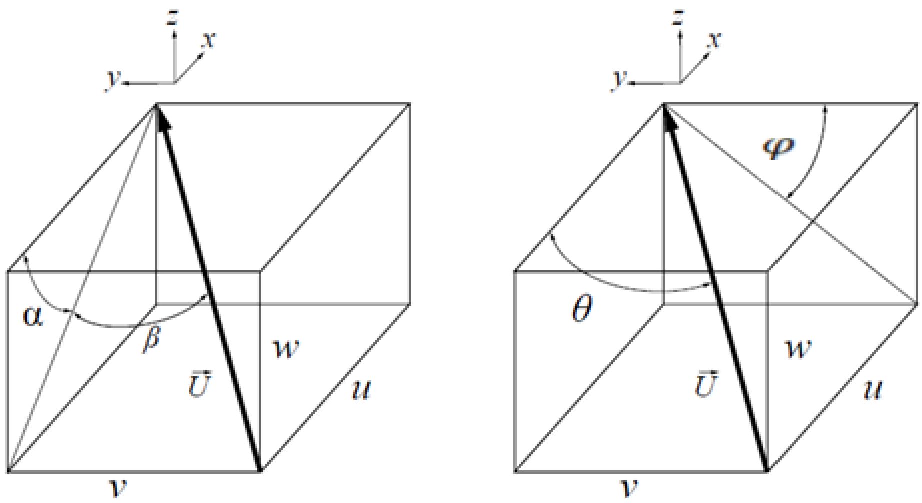

Figure 2 Three components of speed and the related angles [3]

On the picture above it can be seen how these angles correlate with the components of absolute velocity U. Using trigonometry, the two pairs of angles can be correlated as follows [4]:

$$tg(\alpha) = \frac{w}{u}, \tag{1}$$

$$\frac{\sqrt{v^2+w^2}}{u} = tg(\theta), \tag{2}$$

$$\frac{w}{\sqrt{v^2+w^2}} = sin(\varphi), \tag{3}$$

$$\alpha = arctg(tg(\theta)\,sin(\varphi)), \tag{4}$$

Afterwards, knowing the absolute velocity, velocity triangle or the three components of absolute velocity can be calculated using the following equations:

$$U = \frac{v}{cos(\varphi)\,sin(\theta)} = \frac{v}{sin\,(\beta)} \rightarrow \beta = arcsin(sin(\theta)\,cos(\varphi))\,, \tag{5}$$

$$U = \frac{u}{cos(\alpha)\,sin(\beta)} = \frac{u}{cos(\theta)} \rightarrow \theta = arccos(cos(\alpha)\,cos(\beta))\,, \tag{6}$$

$$U\,cos(\beta)\,sin(\alpha) = w, \tag{7}$$

$$U\,sin(\beta)\,tg(\varphi) = w, \tag{8}$$

$$tg(\varphi) = \frac{w}{v}, \tag{9}$$

$$\varphi = arctg(\frac{sin\ (\alpha)}{tg(\beta)}), \tag{10}$$

$$v = U\, sin(\beta). \tag{11}$$

In the equations 6 and 10 it is shown how values of φ and θ are calculated from known values of α and β.

COEFFICIENTS CALCULATION

Probe head can be divided into five sections, one is the central one and two till five are arranged around the one. Each sector has a hole which leads to a pressure sensor. It can be better understood by looking at figure 3.

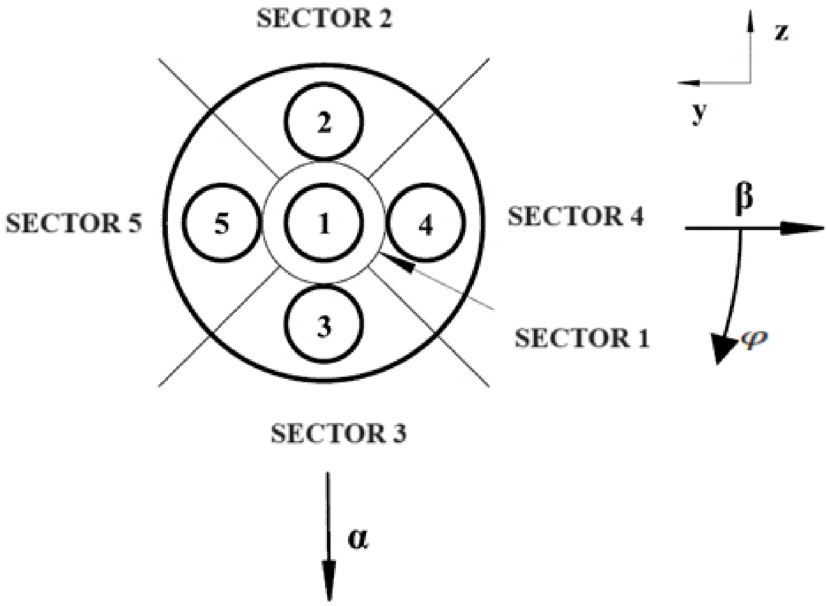

Figure 3 Sections of the head of the FRAP [5]

Depending on the angle of attack, values of the pressures on the sectors change, and some coefficients can be derived from these values. Whether the α and β or φ and θ angles are used, different ways to calculate the coefficients have to be used. α and β angles are used when a narrower angle of attack is considered (under 30˚) and φ and θ are used when a wider angle of attack is considered (from 30˚until the maximum probe operating range of 60˚). For α and β are as follows [4]:

$$b_\alpha = \frac{p_2 - p_3}{p_1 - \bar{p}}, \tag{12}$$

$$b_\beta = \frac{p_5 - p_4}{p_1 - \bar{p}}, \tag{13}$$

$$A_t = \frac{p_1 - p_{tet}}{p_1 - \bar{p}}, \tag{14}$$

$$A_s = \frac{\bar{p} - p_{set}}{p_1 - \bar{p}}, \tag{15}$$

$$\bar{p} = \frac{p_2 + p_3 + p_4 + p_5}{4}, \tag{16}$$

$$p_t = p_1 - A_t(p_1 - \bar{p}), \tag{17}$$

$$p_s = \bar{p} - A_s(p_1 - \bar{p}). \tag{18}$$

And if the φ and θ angles are used, then the equations are as follows [1]:

$$b_\theta = \frac{p_i - p_1}{q}, \tag{19}$$

$$b_\varphi = \frac{p^+ - p^-}{q}, \qquad (20)$$
$$A_t = \frac{p_i - p_{tet}}{q}, \qquad (21)$$
$$A_s = \frac{p_i - p_{set}}{q}, \qquad (22)$$
$$q = p_i - \frac{p^+ - p^-}{2}, \qquad (23)$$
$$p_t = p_i - A_t q, \qquad (24)$$
$$p_s = p_i - A_S q. \qquad (25)$$

p_i is representing the maximum value of pressure from the sectors two till five, p^+ and p^- are then the neighboring sectors where + is in the clockwise direction and – is in the anti-clockwise direction.

In the end partial values of speed are calculated using only the known absolute velocity of the fluid and also the angle of attack:

$$u = U \cdot \cos(\alpha) \cdot \cos(\beta), \qquad (26)$$
$$v = U \cdot \sin(\beta), \qquad (27)$$
$$w = U \cdot \sin(\alpha) \cdot \cos(\beta). \qquad (28)$$

MEASUREMENT UNCERTAINTY

The exact value cannot be determined with absolute certainty, but the goal of measurement equipment and experiments is to arrive at a reliable estimate of the measured value with a certain degree of uncertainty. Measurement uncertainty can be classified into two groups, type A and type B. Type A uncertainty includes estimates obtained through statistical analysis, while type B uncertainty is derived from other methods. In this paper only type A uncertainty will be expanded on [5].

Measurement uncertainty of p_t

When a narrow angle of attack is being worked with (below 30 degrees) then formulae 17 is used to calculate the value of p_t. The fully expanded equation is as follows:

$$p_t = p_1 - A_t(p_1 - \frac{p_2 + p_3 + p_4 + p_5}{4}). \qquad (29)$$

To calculate the combined standard uncertainty of the total pressure, each segment has to be taken into account that goes into calculating the value of total pressure. Which are the values of all pressure sensors of the FRAP and the value of pressure in the Pitot probe. The combined standard uncertainty of any value using its components is calculated using the following equation:

$$\varepsilon_{f(x_1, x_2, \dots, x_N)} = \sqrt{\sum_{i=1}^{N} (\varepsilon_{x_i} \frac{\delta f(x_i)}{\delta x_i})^2}. \qquad (30)$$

If formula 30 is adjusted into the particular case regarding the value of total pressure calculated for a FRAP, then it is going to look like this:

$$\varepsilon_{p_t} = \sqrt{(\varepsilon_{p_1} \frac{\delta p_t}{\delta p_1})^2 + (\varepsilon_{\bar{p}} \frac{\delta p_t}{\delta \bar{p}})^2 + (\varepsilon_{A_t} \frac{\delta p_t}{\delta A_t})^2}. \qquad (31)$$

Now every segment needs to be further expanded so that in the end a complete equation could be obtained which could be used to calculate the value of the uncertainty. The manufacturer states that the probe uses five sensors of the same kind, which are MEGGITT 8507C-1 [3]. This means that they all have the same measurement uncertainty, but their part in the calculation of the combined standard uncertainty of the total pressure are different. Sensor 1 which is located in sector 1 is affecting differently the value of p_t then the other four. Sensors 2 till 5 have the same effect on the calculation of FRAP total pressure, which means that their effect on the systematic uncertainty of it can be summed up and considered as one part. So, the division between the two groups of sensors are sensor 1 as one part and the other, sensors 2 till 5 combined. And the following is gotten:

$$\frac{\delta p_t}{\delta p_1} = 1 - A_t, \tag{32}$$

$$\frac{\delta p_t}{\delta \bar{p}} = A_t, \tag{33}$$

$$\frac{\delta p_t}{\delta A_t} = p_1 - \bar{p}. \tag{34}$$

$$\varepsilon_{\bar{p}} = \frac{\varepsilon_{p_1}}{2} \tag{35}$$

$$\varepsilon_{A_t} = \frac{\varepsilon_{p_1}}{(p_1 - \bar{p})^2}\sqrt{\left(\frac{p_{tet} - \bar{p}}{p_1 - \bar{p}}\right)^2 + \left(\frac{\varepsilon_{p_{tet}}}{\varepsilon_{p_1}}\right)^2 + \left(\frac{1}{2}\frac{p_{tet} - \bar{p}}{p_1 - \bar{p}}\right)^2} \tag{36}$$

When all of the parts which are in the equation 31 are reached, an extended version can be made which is going to be used to calculate the uncertainties. That equation is the following:

$$\varepsilon_{p_t} = \varepsilon_{p_1}\sqrt{(1 - A_t)^2 + \left(\frac{A_t}{2}\right)^2 + \left(\sqrt{\left(\frac{\varepsilon_{p_{tet}}}{\varepsilon_{p_1}}\right)^2 + \frac{5}{4}\left(\frac{p_{tet}-\bar{p}}{p_1-\bar{p}}\right)^2}\right)^2}. \tag{37}$$

Estimation of the systemic uncertainty can be done using experience presented by Town and Camci [9]. The paper suggests that uncertainty from the total pressure coefficient A_t can be calculated using the following equation:

$$\varepsilon_{A_t} = \pm\frac{1}{2}\left(A_{t_{max}} - A_{t_{min}}\right). \tag{38}$$

Where the $A_{t_{max}}$ and $A_{t_{min}}$ are the respective maximum and minimum values of the total pressure coefficient calculated from all the runs.

Measurement uncertainty of the components

Without knowing the measurement uncertainty of particular measurement instruments being used in the measuring chain, the combined standard uncertainty of the desired values cannot be calculated. Particularly in this case, only two types of sensors are being used. One type of sensor is being used inside of the FRAP, and the other is used in the Pitot probe.

Pitot probe measures the total pressure of the stream, and the static pressure is the ambient pressure in the room, the difference of the two is dynamic pressure. Before a pressure sensor is being picked, its operating range has to be firstly calculated. The wind tunnel used to calibrate FRAP has a maximum speed of 60 m/s, but only speeds of less than 40 m/s will be used. Using the formulae for dynamic pressure and that air is used as a fluid, pressure numbers of less than 1000 Pa are obtained. This leads to the choosing of a Testo 0638 1447 pressure sensor, which has an operating range of between 0 and 1000 Pa, with the accuracy of 3 Pa [6].

As previously mentioned all five sensors inside of the FRAP are piezoresistive pressure transducers MEGGITT 8507C-1. The manufacturer does not give out the value of the measurement uncertainty like Testo, so it has to be calculated. Here are the individual uncertainties:

Table 1 Individual uncertainties [7]

Type of uncertainty	*Symbol*	*Value* [%]
Pressure hysteresis total	hyst tot	1.5
Non-linearity, independent	non-lin	1
Non-repeatability	non-rep	0.1
Pressure hysteresis	hyst	0.1
Zero shift	zero	0.2
Span shift	span	0.2

According to JCGM 100:2008 the formulae for the calculating the combined standard uncertainty of a sensor is the following [8]:

$$\varepsilon_{p_{p_1}} = \sqrt{\varepsilon_{p_{zero}}{}^2 + \varepsilon_{p_{span}}{}^2 + \varepsilon_{p_{hyst}}{}^2 + \varepsilon_{p_{non-lin}}{}^2 + \varepsilon_{p_{non-rep}}{}^2}, \quad (39)$$

$$\varepsilon_{p_{p_1}} = \sqrt{\varepsilon_{p_{zero}}{}^2 + \varepsilon_{p_{span}}{}^2 + \varepsilon_{p_{hyst\,tot}}{}^2}. \quad (40)$$

Depending if it is chosen to use Pressure hysteresis as a total value, or to choose to use its components separately then formulas 40 and 39 are used respectively. If equation 39 is used, then $\varepsilon_{p_{p_1}}$ of 1.048% is obtained, and if 40 is used then $\varepsilon_{p_{p_1}}$ of 1.526% is obtained. The difference between the two is almost 50% which is a large enough difference for the user to pick any of the two. So, the larger combined standard uncertainty value is adopted and is used further on. All values of uncertainties represent standard uncertainties.

The whole calculation of the uncertainty is done without taking into consideration the uncertainties which arrive from the acquisition equipment. In this case it is the National Instruments card. This is done purposely because, the card which is going to be used in the calibration and in further work is a 24-bit card, which can only improve the resolution of the results. Thereby improving the uncertainty of the system.

CONCLUSION

Accurate measurements are an ideal that may never be fully realized, but striving for precision remains a fundamental goal for researchers and scientists. Understanding and quantifying uncertainties is crucial, as it ensures that all measurements are interpreted within the context of their limitations. The work presented in this paper is focused on calculating the uncertainty due to systematic errors of a Fast Response Aerodynamic Probe (FRAP). By meticulously determining these uncertainties, this research not only enhances the reliability of FRAP but also paves the way for its broader application in future scientific studies, particularly in the fields of stream flow and turbulence analysis. The findings of this work contribute to the refinement of methodologies, ensuring that future experiments can be conducted with greater confidence in the accuracy and precision of their results. Furthermore, this research underscores the importance of uncertainty analysis in the development of robust experimental techniques, highlighting its role in advancing scientific knowledge. By establishing a framework for evaluating and minimizing uncertainties, this study sets a standard for future work in aerodynamics and fluid dynamics, ultimately leading to more reliable and insightful conclusions in these critical areas of research.

NOMENCLATURE

α – rotation angle in right plane,

β – rotation angle around the handle,

φ – rotation angle in frontal plane,

θ – conic angle,

U – total velocity of fluid in the measured cross-section [m/s],

u – x speed component [m/s],

v – y speed component [m/s],

w – z speed component [m/s],

b_α –pressure coefficient for angle α,

b_β – pressure coefficient for angle β,

A_t – total pressure coefficient,

A_s – static pressure coefficient,

q – pseudo dynamic pressure,

p_i – pressure in i hole [Pa],

p_t – total fluid pressure [Pa],

p_{tet} – total fluid pressure of Pitot probe (etalon probe) [Pa]

p_{set} – static fluid pressure given by barometer (etalon device) [Pa]

p_s – static fluid pressure [Pa],

b_φ – pressure coefficient for angle φ,

b_θ – pressure coefficient for angle θ,

p^+ – pressure inside of the hole clock-wise in correspondence to the hole with the highest value of pressure [Pa],

p^- – pressure inside of the hole anti clock-wise in correspondence to the hole with the highest value of pressure [Pa],

$\varepsilon_{f(x_1,x_2,\ldots,x_N)}$ – the combined standard uncertainty of a function $f(x_1, x_2, \ldots, x_n)$,
ε_{p_t} – uncertainty due to systematic errors of total pressure of FRAP,
ε_{A_t} – uncertainty due to systematic errors of total pressure coefficient of FRAP.

ACKNOWLEDGEMENTS

The results presented in this paper are the result of the research supported by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia under the Agreement on financing the scientific research work of teaching staff at accredited higher education institutions in 2024, no. 451-03-65/2024-03/200105 dated on February 5th, 2024.

REFERENCES

[1] Heckmeier F. M., Kienitz S., Iglesias D., Breitsamter C. (2019): An Innovative Development of a Five Hole Pressure Probe for Highly Unsteady Flow Phenomena Proceedings of ASME Turbo Expo 2019, Turbomachinery Technical Conference and Exposition, June 2019, Vol 6, Phoenix, Arizona, USA, ASME, DOI:10.1115/GT2019-90079

[2] Brouckaert J. F. (2007): Fast Response Aerodynamic Probes for Measurements in Turbomachines, von Karman Institute for Fluid Dynamics, Journal of Power and Energy, September 2007, Vol 221, Issue 6, pp 803-813, DOI: 10.1243/09576509JPE460

[3] https://vectoflow.de/en/ - Website of Manufacturer of Probes и Vectorflow GMBH, Visited on 06.06.2024.

[4] Johansen E. S., Rediniotis O. K., Jones G. (2001): The Compressible Calibration of Miniature Multi-Hole Probes, Journal of Fluids Engineering, Vol. 123, Issue 1, pp 128-138, March 2001, ASME, DOI: 10.1115/1.1334377

[5] Bulajić M. Z. (2024): System for Calibrating Spacial Probes for Measuring Speed and Pressure, Master's thesys, Faculty of Mechanical Engineering, University of Belgrade, Belgrade, Serbia

[6] https://www.testo-direct.com/product/testo-0638-1447-differential-pressure-probe - Website of Manufacturer of Sensors TESTO

[7] https://akron.be/wp-content/uploads/2018/04/Endevco-8507C.pdf - PDF from the Manufacturer Endevco

[8] JCGM 100:2008, Evaluation of measurement data — Guide to the expression of uncertainty in measurement, 2008.

[9] Town J., Camci C. (2011): Sub-Miniature Five-Hole Probe Calibration Using a Time Efficient Pitch and Yaw Mechanism and Accuracy Improvements, Proceedings of the ASME 2011Turbo Expo, Vol 3, pp. 349-359, DOI: 10.1115/GT2011-46391

[10] WMO-No. 8, Guide to Instruments and Methods of Observation, 2021

[11] ISO 17713-1, Wind tunnel test methods for rotating anemometer performance, Geneva, Switzerland, 2007

[12] ISO 3966, Measurement of fluid flow in closed conduits — Velocity area method using Pitot static tubes, Geneva, Switzerland, 2020

AUTHORS

Milan Z. Bulajić, MSc. Mechanical Engineering, e-mail: milan.bulajicvkp@gmail.com , mob.: 060/5850-204

Prof. Dr. Đorđe S. Čantrak, University of Belgrade-Faculty of Mechanical Engineering, Hydraulic Machinery and Energy Systems Department, Address: Kraljice Marije 16, 11120 Belgrade 35, Serbia, e-mail: djcantrak@mas.bg.ac.rs, tel.: 011/3302-226, mob.: 063/8323-752

Assoc. Prof. Dr. Dejan B. Ilić, , University of Belgrade-Faculty of Mechanical Engineering, Hydraulic Machinery and Energy Systems Department, Address: Kraljice Marije 16, 11120 Belgrade 35, Serbia, e-mail: dilic@mas.bg.ac.rs, mob.: 063/8326-432